# Calc II 10.1

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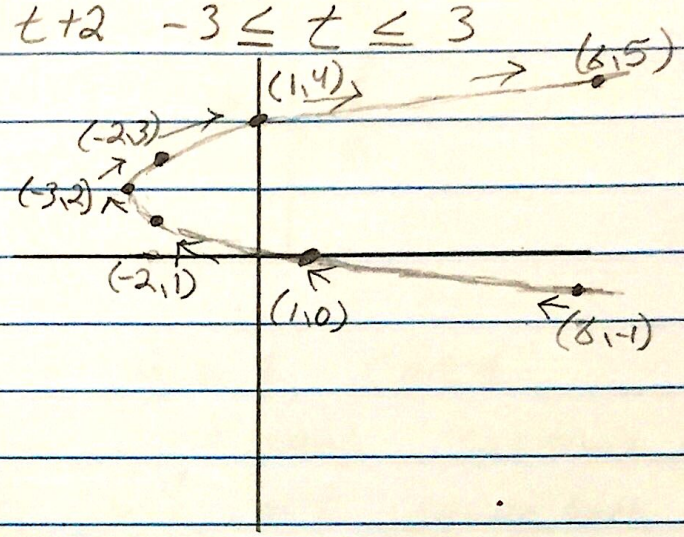
Problem # 7

(a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases. (b) Eliminate the parameter to find a Cartesian equation of the curve.

$x = t^2 - 3 \quad y = t + 2 \quad -3 \le t \le 3$

| t | x | y |
|---|---|---|
| -3 | 6 | -1 |
| -2 | 1 | 0 |
| -1 | -2 | 1 |
| 0 | -3 | 2 |
| 1 | -2 | 3 |
| 2 | 1 | 4 |
| 3 | 6 | 5 |




so curve depicts a parabola that would pass the horizontal line test, so we can find a cartesian equation of the curve. As anything in math what starts out as something fun and sweet it quickly becomes bitter and painful

$y = t + 2$ so $t = y - 2$ by subtracting 2 from both sides

$x = t^2 - 3$ so in the place of t we substitute $y - 2$ thus $x = t^2 - 3$

becomes $x = (y-2)^2 - 3$ Foil $(y-2)(y-2)$ so $x = [y^2 - 2y - 2y + 4] - 3$

Foil gives $\rightarrow x = [y^2 - 2y - 2y + 4] - 3$

$x = y^2 - 4y + 4 - 3$

the equation of the curve $\rightarrow$ $x = y^2 - 4y + 1$

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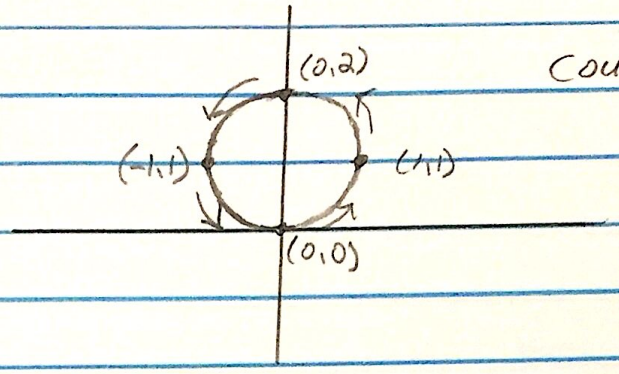
Problem # 8 $x = \sin t$ $y = 1 - \cos t$ $0 \le t \le 2\pi$

| $t$ | $x$ | $y$ |
|---|---|---|
| $0$ | $0$ | $0$ |
| $\frac{\pi}{2}$ | $1$ | $1$ |
| $\pi$ | $0$ | $2$ |
| $\frac{3\pi}{2}$ | $-1$ | $1$ |
| $2\pi$ | $0$ | $0$ |




counter clockwise

$x = \sin t$ $y = 1 - \cos t$

square both sides $x^2 = \sin^2 t$ $y - 1 = -\cos t$ subtract one from both sides

$(y-1)^2 = \cos^2 t$ square both sides

So $x^2 + (y-1)^2 = \sin^2 t + \cos^2 t$

So the cartesian equation is $x^2 + (y-1)^2 = 1$

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Problem #1

Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as $t$ increases

$$x = 1 - t^2, \quad y = 2t - t^2, \quad -1 \le t \le 2$$

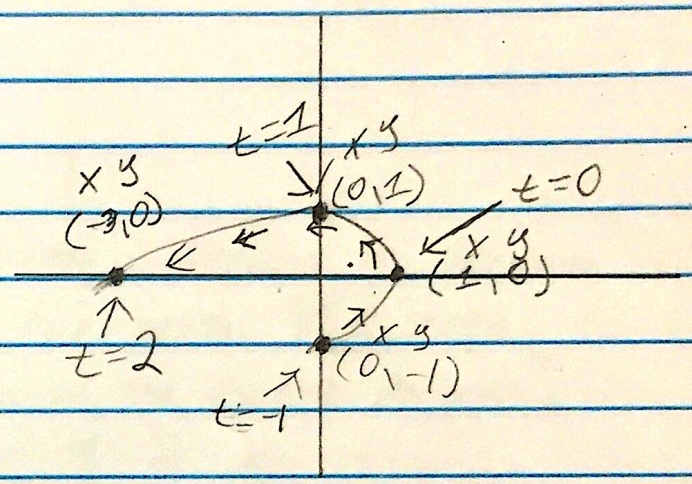
$x_{-1} = 1 - (-1)^2 = 0 \qquad y_{-1} = 2(-1) - (-1)^2 = -1$

| $t$ | $X$ | $Y$ |
|---|---|---|
| -1 | 0 | -1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 2 | -3 | 0 |

$x_0 = 1 - (0)^2 = 1 \qquad y_0 = 2(0) - 0^2 = 0$

$x_1 = 1 - (1)^2 = 0 \qquad y_1 = 2(1) - 1^2 = 1$

$x_2 = 1 - (2)^2 = -3 \qquad y_2 = 2(2) - (2)^2 = 0$




So we defined the curve and its direction.

So in this parametric equation if we were to say it represented the motion of a particle It would tell us when the particle was at a point, and the direction of the motion of the particle.

## 10.1 Curves Defined by Parametric Equations

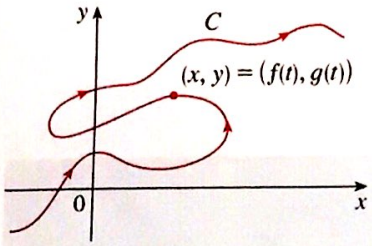



**FIGURE 1**

Imagine that a particle moves along the curve $C$ shown in Figure 1. It is impossible to describe $C$ by an equation of the form $y = f(x)$ because $C$ fails the Vertical Line Test. But the $x$- and $y$-coordinates of the particle are functions of time and so we can write $x = f(t)$ and $y = g(t)$. Such a pair of equations is often a convenient way of describing a curve and gives rise to the following definition.

Suppose that $x$ and $y$ are both given as functions of a third variable $t$ (called a **parameter**) by the equations

$$x = f(t) \qquad y = g(t)$$

(called **parametric equations**). Each value of $t$ determines a point $(x, y)$, which we can plot in a coordinate plane. As $t$ varies, the point $(x, y) = (f(t), g(t))$ varies and traces out a curve $C$, which we call a **parametric curve**. The parameter $t$ does not necessarily represent time and, in fact, we could use a letter other than $t$ for the parameter. But in many applications of parametric curves, $t$ does denote time and therefore we can interpret $(x, y) = (f(t), g(t))$ as the position of a particle at time $t$.

**EXAMPLE 1** Sketch and identify the curve defined by the parametric equations

$$x = t^2 - 2t \qquad y = t + 1$$

**SOLUTION** Each value of $t$ gives a point on the curve, as shown in the table. For instance, if $t = 0$, then $x = 0$, $y = 1$ and so the corresponding point is $(0, 1)$. In Figure 2 we plot the points $(x, y)$ determined by several values of the parameter and we join them to produce a curve.

| $t$ | $x$ | $y$ |
|---|---|---|
| −2 | 8 | −1 |
| −1 | 3 | 0 |
| 0 | 0 | 1 |
| 1 | −1 | 2 |
| 2 | 0 | 3 |
| 3 | 3 | 4 |
| 4 | 8 | 5 |

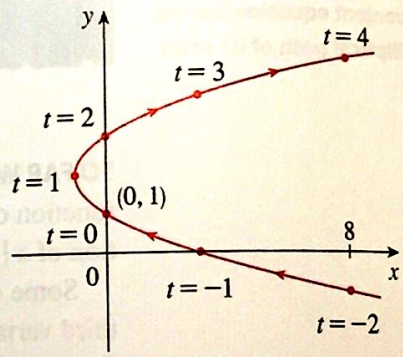



**FIGURE 2**

A particle whose position is given by the parametric equations moves along the curve in the direction of the arrows as $t$ increases. Notice that the consecutive points marked on the curve appear at equal time intervals but not at equal distances. That is because the particle slows down and then speeds up as $t$ increases.

It appears from Figure 2 that the curve traced out by the particle may be a parabola. This can be confirmed by eliminating the parameter $t$ as follows. We obtain $t = y - 1$ from the second equation and substitute into the first equation. This gives

$$x = t^2 - 2t = (y - 1)^2 - 2(y - 1) = y^2 - 4y + 3$$

and so the curve represented by the given parametric equations is the parabola $x = y^2 - 4y + 3$.

This equation in $x$ and $y$ describes *where* the particle has been, but it doesn't tell us *when* the particle was at a particular point. The parametric equations have an advantage—they tell us *when* the particle was at a point. They also indicate the *direction* of the motion.

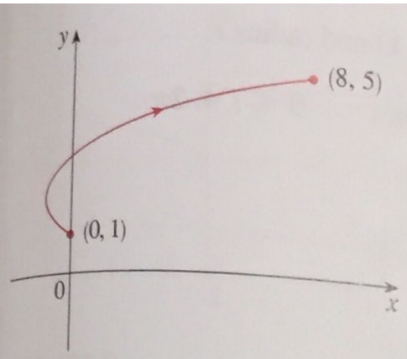



FIGURE 3

No restriction was placed on the parameter $t$ in Example 1, so we assumed that $t$ could be any real number. But sometimes we restrict $t$ to lie in a finite interval. For instance, the parametric curve

$$x = t^2 - 2t \qquad y = t + 1 \qquad 0 \leqslant t \leqslant 4$$

shown in Figure 3 is the part of the parabola in Example 1 that starts at the point (0, 1) and ends at the point (8, 5). The arrowhead indicates the direction in which the curve is traced as $t$ increases from 0 to 4.

In general, the curve with parametric equations

$$x = f(t) \qquad y = g(t) \qquad a \leqslant t \leqslant b$$

has **initial point** $(f(a), g(a))$ and **terminal point** $(f(b), g(b))$.

**EXAMPLE 2** What curve is represented by the following parametric equations?

$$x = \cos t \qquad y = \sin t \qquad 0 \leqslant t \leqslant 2\pi$$

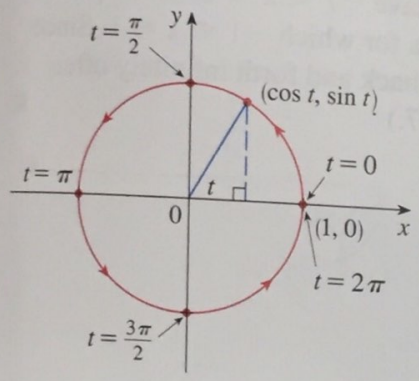



FIGURE 4

**SOLUTION** If we plot points, it appears that the curve is a circle. We can confirm this impression by eliminating $t$. Observe that

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

Thus the point $(x, y)$ moves on the unit circle $x^2 + y^2 = 1$. Notice that in this example the parameter $t$ can be interpreted as the angle (in radians) shown in Figure 4. As $t$ increases from 0 to $2\pi$, the point $(x, y) = (\cos t, \sin t)$ moves once around the circle in the counterclockwise direction starting from the point (1, 0). ■

**EXAMPLE 3** What curve is represented by the given parametric equations?

$$x = \sin 2t \qquad y = \cos 2t \qquad 0 \leqslant t \leqslant 2\pi$$

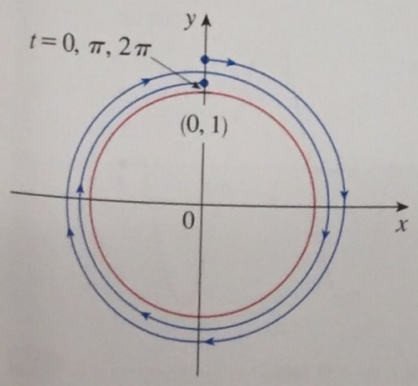



FIGURE 5

**SOLUTION** Again we have

$$x^2 + y^2 = \sin^2 2t + \cos^2 2t = 1$$

so the parametric equations again represent the unit circle $x^2 + y^2 = 1$. But as $t$ increases from 0 to $2\pi$, the point $(x, y) = (\sin 2t, \cos 2t)$ starts at (0, 1) and moves *twice* around the circle in the clockwise direction as indicated in Figure 5. ■

Examples 2 and 3 show that different sets of parametric equations can represent the same curve. Thus we distinguish between a *curve*, which is a set of points, and a *parametric curve*, in which the points are traced in a particular way.

**EXAMPLE 4** Find parametric equations for the circle with center $(h, k)$ and radius $r$.

**SOLUTION** If we take the equations of the unit circle in Example 2 and multiply the expressions for $x$ and $y$ by $r$, we get $x = r\cos t$, $y = r\sin t$. You can verify that these equations represent a circle with radius $r$ and center the origin traced counterclockwise. We now shift $h$ units in the $x$-direction and $k$ units in the $y$-direction and obtain para-